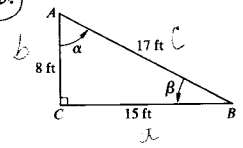
Exercises 5.2

[09-09-29-MT-11]

1. Show that the way of expressing $\tan\alpha$ in this section is consistent with the definition of the tangent function.
2. Show that the way of expressing $\sec\alpha$ in this section is consistent with the definition of the secant function.
3. Show that the way of expressing $\csc\alpha$ in this section is consistent with the definition of the cosecant function.
4. Show that the way of expressing $\cot\alpha$ in this section is consistent with the definition of the cotangent function.

In each of the problems 5–10 give the six trigonometric functions of α and β, first as fractions and then as decimals. Then use the given information to determine α and β to the nearest minute.

5.



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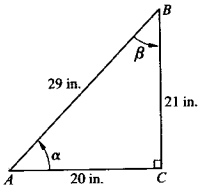

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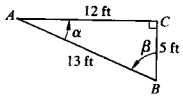

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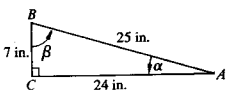

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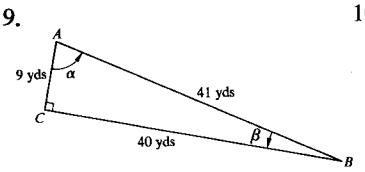
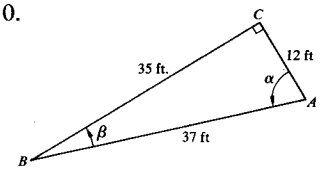


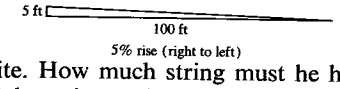
10.



In each of the following problems use the given information to solve the triangle. Assume the given information is exact.

11. $\gamma = 90°$, $a = 3$ ft, $b = 3$ ft.
12. $\gamma = 90°$, $a = 8$ in. $b = 16$ in.
13. $\gamma = 90°$, $a = 3$ yds, $c = 6$ yds.
14. $\gamma = 90°$, $a = 4$ ft, $c = 7$ ft.
15. $\gamma = 90°$, $b = 7$ in., $c = 10$ in.
16. $\gamma = 90°$, $b = 13$ yds, $c = 20$ yds.
17. $\gamma = 90°$, $a = 5$ ft, $\alpha = 27°$.
18. $\gamma = 90°$, $a = 7$ in., $\alpha = 51°20'$.
19. $\gamma = 90°$, $b = 15$ yds, $\beta = 38°43'$.
20. $\gamma = 90°$, $b = 20$ ft, $\beta = 81°9'$
21. $\gamma = 90°$, $a = 5$ in., $\beta = 27°$.
22. $\gamma = 90°$, $a = 7$ yds, $\beta = 57°20'$.
23. $\gamma = 90°$, $b = 15$ ft, $\alpha = 62°27'$.
24. $\gamma = 90°$, $b = 20$ in., $\alpha = 19°14'$.

25. Find the height of a vertical tree that casts a shadow 85 feet long on level ground if the angle of elevation of the sun is 23°.
26. How long a shadow does a 6-foot-tall man cast on level ground if the angle of elevation of the sun is 40°?
27. A vacant rectangular lot is 80 feet long and 50 feet wide. A path has been trod which is a diagonal for the rectangular lot. Find the length of the path and the angle it makes with each of the adjacent sides of the lot.
28. Find the lengths of the congruent sides of an isosceles triangle if the length of the base is 12 inches and the measure of the angle opposite the base is 40°.
29. A 15-foot ladder is placed against the side of a building so that the top of the ladder is 13 feet above the ground. If the ground is level, how far is the base of the ladder from the building? What angle does the ladder make with the building?
30. A road is said to have a 5 per cent grade if the road rises 5 feet while moving forward 100 feet horizontally. How many feet does a 7 per cent road rise if the length of the road is 7 miles? (*Hint:* the road is the hypotenuse).



31. A boy is flying a kite. How much string must he have to get the kite 150 feet above the ground if the string makes an angle of 25° with the ground?

Exercises 5.2

Use Figure 5.13 for 1 and 3.

1. $\tan\alpha = \frac{a}{b} = \left(\frac{a}{c}\right) \div \left(\frac{b}{c}\right) = \frac{\sin\alpha}{\cos\alpha} = \frac{\sin x}{\cos x} = \tan x.$

3. $\csc\alpha = \frac{c}{a} = 1 \div \left(\frac{a}{c}\right) = \frac{1}{\sin\alpha} = \frac{1}{\sin x} = \csc x$

5. $\sin\alpha = \frac{15}{17} = .882 = \cos\beta.$ $\quad \cos\alpha = \frac{8}{17} = .471 = \sin\beta.$

   $\tan\alpha = \frac{15}{8} = 1.875 = \cot\beta.$ $\quad \cot\alpha = \frac{8}{15} = .5333 = \tan\beta.$

   $\sec\alpha = \frac{17}{8} = 2.125 = \csc\beta.$ $\quad \csc\alpha = \frac{17}{15} = 1.133 = \sec\beta.$

7. $\sin\alpha = \frac{5}{13} = .385 = \cos\beta.$ $\quad \cos\alpha = \frac{12}{13} = .923 = \sin\beta.$

   $\tan\alpha = \frac{5}{12} = .417 = \cot\beta.$ $\quad \cot\alpha = \frac{12}{5} = 2.4 = \tan\beta.$

   $\sec\alpha = \frac{13}{12} = 1.083 = \csc\beta.$ $\quad \csc\alpha = \frac{13}{5} = 2.6 = \sec\beta.$

9. $\sin\alpha = \frac{40}{41} = .976 = \cos\beta.$ $\quad \cos\alpha = \frac{9}{41} = .220 = \sin\beta.$

   $\tan\alpha = \frac{40}{9} = 4.444 = \cot\beta.$ $\quad \cot\alpha = \frac{9}{40} = .225 = \tan\beta.$

   $\sec\alpha = \frac{41}{9} = 4.556 = \csc\beta.$ $\quad \csc\alpha = \frac{41}{40} = 1.025 = \sec\alpha.$

11. $\alpha = 45°$, $\beta = 45°$, $c = 3\sqrt{2}$ ft.
13. $\alpha = 30°$, $\beta = 60°$, $b = 3\sqrt{3}$ yds.
15. $\alpha = 45°34'$, $\beta = 44°26'$, $a = 7.142$ in.
17. $\beta = 63°$, $b = 9.815$ ft, $c = 11.015$ ft.
19. $\alpha = 51°17'$, $a = 18.63$ yds, $c = 23.985$ yds.
21. $\alpha = 63°$, $b = 2.548$ in., $c = 5.610$ in.
23. $\beta = 27°33'$, $a = 28.76$ ft, $c = 32.43$ ft.
25. 36.1 ft
27. 94.3 ft, 58°, 32°
29. 7.48 ft, 29°56'
31. 355 ft